Reasoning under Uncertainty
+ FOL Normalization & Skolemization

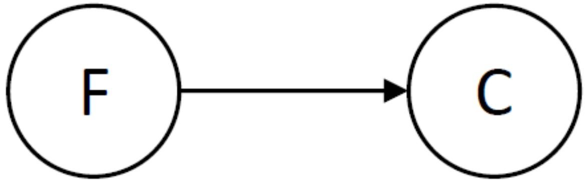
A TUTORIAL

INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

1. **Consider the following Bayesian network, where F = having the flu and C =coughing:**

a) **Write down the joint probability table specified by the Bayesian network.**

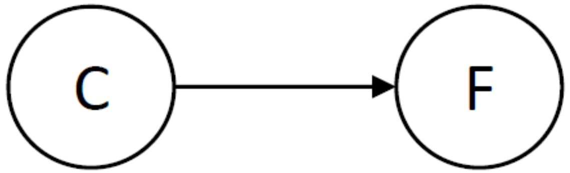
$P(F) = 0.1$



$P(C \mid F) = 0.8$

$P(C \mid \neg F) = 0.3$

b) **Determine the probabilities for the following Bayesian network so that so that it species the same joint probabilities as the given one.**



c) **Are C and F independent in the Bayesian network of Part a?**

d) **Are C and F independent in the Bayesian network of Part b?**

Solution 1:

a)

F	C	
t	t	$0.1 \times 0.8 = 0.08$
t	f	$0.1 \times 0.2 = 0.02$
f	t	$0.9 \times 0.3 = 0.27$
f	f	$0.9 \times 0.7 = 0.63$

b) P(C) = 0.08 + 0.27 = 0.35

P(F|C) = P(F, C) / P(C) = 0.08 / 0.35 ~ 0.23

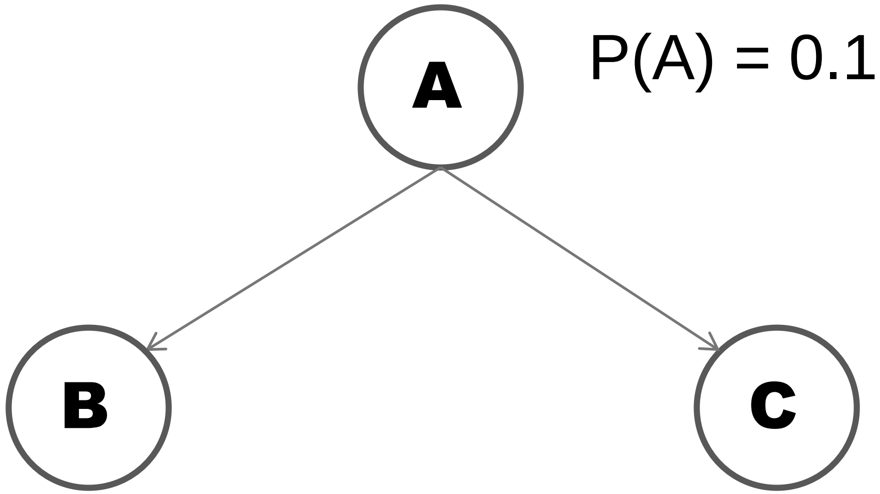
P(F|C') = P(F, C') / P(C') = 0.02/0.65 ~ 0.03

C' -> NOT(C)

c) No; because P(C) = 0.35 but P(C | F) = 0.8

d) No; for the same reason.

2. Consider the following Bayesian network:



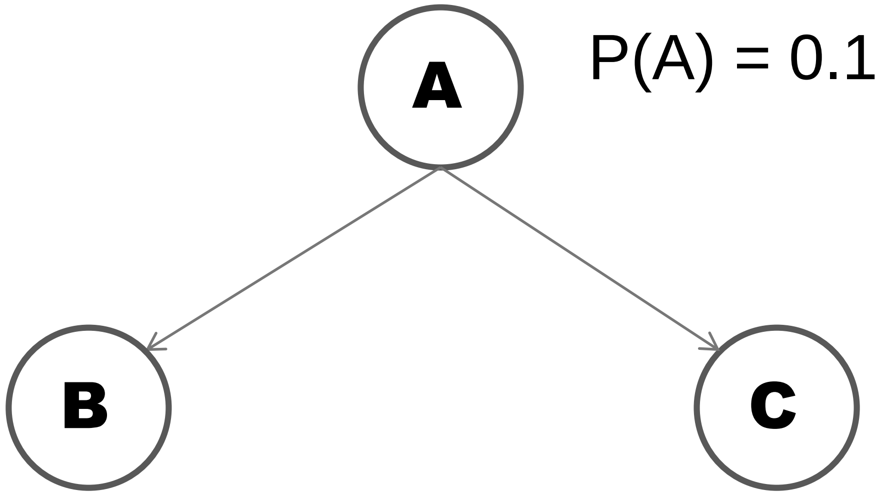
P(A) = 0.10664

A	P(B)
T	0.85
F	0.04

A	P(C)
T	0.75
F	0.1

(a) Compute P(A | B’, C)

1. **Consider the following Bayesian network:**



P(A) = 0.10664

A	P(B)
T	0.85
F	0.04

A	P(C)
T	0.75
F	0.1

(b) Now add on to the network above a fourth node containing Boolean random variable D, with arcs to it from both B and C. Answer the following Yes/No questions with reasons:

1. **Is A conditionally independent of D given B?**
2. **Is B conditionally independent of C given A?**

Solution 2:

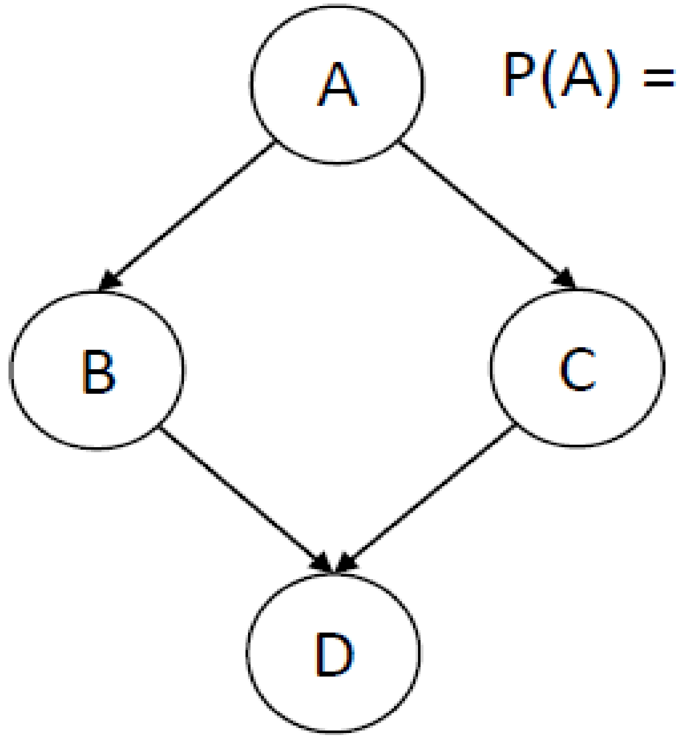
a) P(A | B’, C) = P(B’, C | A) P(A) / P(B’, C)
= P(B’, C | A) P(A) / (P(B’, C | A) P(A) + P(B’, C | A’) P(A’)) ------------- (1)

Now, P (B', C | A) = P (B' | A) P (C | A) = (0.15)(0.75) = 0.1125

So, From (1) we have, P(A | B’, C)
= (0.1125 * 0.10664) / ((0.1125 * 0.10664) + (0.096* 0.89336))
= 0.1227

b) i. No (An undirected path between A and D exists through C)
ii. Yes (All paths either pass through A which is an evidence node, or D which is not an evidence node, but has all edges as incoming edges)

3. Consider the following Bayesian network. A, B, C, and D are Boolean random variables. If we know that A is true, what is the probability of D being true?



$P(A) = 0.75$

$P(B \mid A) = 0.2$

$P(B \mid \neg A) = 0.5$

$P(C \mid A) = 0.7$

$P(C \mid \neg A) = 0.25$

$P(D \mid B \wedge C) = 0.3$

$P(D \mid B \wedge \neg C) = 0.25$

$P(D \mid \neg B \wedge C) = 0.1$

$P(D \mid \neg B \wedge \neg C) = 0.35$

Solution 3:

$P(D|A) = P(A, D) / P(A)$

$= (P(A, B, C, D) + P(A, B, \neg C, D) + P(A, \neg B, C, D) + P(A, \neg B, \neg C, D)) / P(A)$

$= P(B \mid A)\, P(C \mid A)\, P(D \mid B, C) + P(B \mid A)\, P(\neg C \mid A)\, P(D \mid B, \neg C) +$

$P(\neg B \mid A)\, P(C \mid A)\, P(D \mid \neg B, C) + P(\neg B \mid A)\, P(\neg C \mid A)\, P(D \mid \neg B, \neg C)$

$= (0.2 \times 0.7 \times 0.3) + (0.2 \times 0.3 \times 0.25) + (0.8 \times 0.7 \times 0.1) + (0.8 \times 0.3 \times 0.35)$

$= 0.042 + 0.015 + 0.056 + 0.084$

$= 0.197$

Skolemization and Normalization in FOL: Example 1

Every philosopher writes at least one book.

$$\forall x[\text{Philo}(x) \rightarrow \exists y[\text{Book}(y) \wedge \text{Write}(x, y)]]$$

Main Steps:

Eliminate Implication: $\forall x[\neg \text{Philo}(x) \vee \exists y[\text{Book}(y) \wedge \text{Write}(x, y)]]$

Skolemize: ***Substitute y by g(x)***

$$\forall x[\neg \text{Philo}(x) \vee [\text{Book}(g(x)) \wedge \text{Write}(x, g(x))]]$$

Skolemization and Normalization in FOL: Example 2

All students of a philosopher read one of their teacher's books.

$$\forall x \forall y[Philo(x) \wedge StudentOf(y, x) \rightarrow \exists z[Book(z) \wedge Write(x, z) \wedge Read(y, z)]]$$

Main Steps:

Eliminate Implication:

$$\forall x \forall y[\neg Philo(x) \vee \neg StudentOf(y, x) \vee \exists z[Book(z) \wedge Write(x, z) \wedge Read(y, z)]]$$

Skolemize: ***Substitute z by h(x, y)***

$$\forall x \forall y[\neg Philo(x) \vee \neg StudentOf(y, x) \vee [Book(h(x, y)) \wedge Write(x, h(x, y)) \wedge Read(y, h(x, y))]]$$

Skolemization and Normalization in FOL: Example 3

There exists a philosopher with students.

$$\exists x \exists y[Philo(x) \land StudentOf(y, x)]$$

Main Steps:

Skolemize: ***Substitute x by a and y by b***

$$Philo(a) \land StudentOf(b, a)$$